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A New Method for Solving Bi-Objective Fractional Transportation Problems

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Abstract

A new method is proposed for finding a set of efficient solutions to bi-objective fractional transportation problems. This method is an important tool for the decision makers to obtain efficient solutions and select the preferred optimal solution from the satisfaction level. The procedure allows the user to identify next efficient solution to the problem from the current efficient solution, which differs from utility function method, goal programming approach, genetic approach and evolutionary approach. This new approach enables the decision makers to evaluate the economic activities and make satisfactory managerial decisions when they are handling a variety of logistic problems involving two objectives. An illustrative example is presented to clarify the idea of the proposed approach.

Keywords: Bi-Objective Fractional Transportation Problems, Efficient Solution, Linear Fractional Programming Problems, Level of Satisfaction.

Introduction

Transportation problem nourishes economic and social activity and is cardinal to operations research and management science. In the classical transportation problem of linear progra mming, the traditional objective is one of minimizing the total cost multi-objective transportation problem and linear fractional programming problem has attracted the attention of many researchers in the past. In general, the real life problems are modeled with multi-objectives which are measured in different scales and at the same time in conflict. In actual classical transportation problem s, the multi-objective functions are generally considered, which includes average delivery time of the commodities, reliability of transportation, product deterioration and so on. A number of optimization problems are actually multi-objective optimization problems (MOPs), where the objectives are conflicting. As a result, there is usually no single solution which optimizes all objectives simultaneously. A number of techniques have been developed to find a compromise solution to MOPs. The reader is referred to the recent book by Miettinen (Miettinen, 1999) about the theory and algorithms for MOPs. Fractional programming problems (FPPs) arise from many applied areas such as portfolio selection, stock cutting, game theory, and numerous decision problems in management science. Many approaches for FPPs have been exploited in considerable details. See, for example, Bitran and Novaes (Bitran & Novaes, 1973), A. Charnes and Cooper (Charnes & Cooper, 1962), Craven (Craven, 1988), Schaible (Schaible, 1995), Schaible and Ibaraki (Schaible & Ibaraki, 1983) and Stancu-Minasian (Stancu , 1997) Multi-objective Linear fractional programming problems useful targets in production and financial planning and return on investment.There are several ways to solving the linear fractional programming (LFP) and multi-objective linear fractional programming (MOLFP) problems (Borza et al., 2012; M.Chakraborty, 2002; Chankong & Haimes, 1983) Tantawy Proposed a new method for solving linear fractional programming problems (Tantawy, 2007). Singh, Sharma and Dangwal Proposed a solution concept to MOLFP problem using the Taylor polynomial series at optimal point of each linear fractional function in feasible region (angwal, 2012) Sulaiman and Abulrahim Used transformation technique for solving multiobjective linear fractional programming problems to single objective linear fractional programming problem through a new method using mean and median and then solve the problem by modified simplex method (Sulaiman & Abdulrahim, 2013; Sulaiman et al., 2014). Bodkhe et al., (2010) used the fuzzy programming technique with hyperbolic membership function to solve a bi-objective TP as vector minimum problem (Bodkhe et al., 2010). Pandian and Natarajan, (2010) have introduced the zero point method for finding an optimal solution to a classical transportation problem without using any optimality checking methods (Pandian & Natarajan, 2010). Sheikhi, (2014) have introduced a novel algorithm for solving two-objective fuzzy transportation problems (Sheikhi, 2014).

In this paper, we propose a new method namely, Determine the set of efficient solutions to bi-objective transportation fractional problem. In the proposed method, we can identify next solution to the problem from the current solution which differs from utility function method, goal programming approach, fuzzy programming technique, genetic approach and evolutionary approach. The percentage level of satisfaction of a solution of the bi-objective transportation fractional problem is introduced. This

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method is illustrated with help of a numerical example. This new approach enables the decision makers to evaluate the economical activities and make self- satisfied managerial decisions when they are handling a variety of logistic problems involving two objectives.

*The Fractional Transportation Simplex Method (*Bajalinov, 2012)

As in the case of a general linear fractional programming problem, the solution process of a linear fractional transportation problem (LFTP) consists of two phases:

1. Finding an initial basic feasible solution (BFS);
2. Improving the current basic feasible solution until the optimality criterion is satisfied.

Since the process of finding initial BFS for LFPT is the same as in the linear problem (LP) case, we will focus mainly on the second stage.
Consider the following LFPT problem:

$$\text{(LFTP)} \quad \text{Max } Q(x) = \frac{P(x)}{D(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}x_{ij} + p_0}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij} + d_0}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad for\ \ i1,2,\dots,m \qquad (2.1)$$

$$\sum_{i=1}^{n} x_{ij} = b_j \quad for\ \ j = 1,2,\dots,n \qquad (2.2)$$

$$x_{ij} \geq 0, \ \ i = 1,2,\dots,m; j = 1,2,\dots,n \qquad (2.3)$$

Here and in what follows we suppose that $D(x) > 0$, $\forall x = (x_{ij}) \in S$, where, **S** denotes a feasible set defined by constraints (2.1)-(2.3). Further, we assume that $a_i > 0,\ b_j > 0, i = 1,2,\dots,m; j = 1,2,\dots,n$ and total demand equals to total supply, i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

We now show how the simplex method may be adapted to the case when an LFPT problem is to be solved. First, we have to introduce special simplex multipliers u_i', v_j' and u_i'', v_j'' associated with numerator P(X) and denominator D(x), respectively. Elements u_i' and u_i'', $i = 1,2, \dots, m$, correspond to m supply constraints and elements v_j' *and* v_j'', $j = 1,2,\dots,n$, correpond to n demand constraints. We calculate these variables from the following systems of linear eq- -uations

$$u_i' + v_j' = p_{ij} \quad \text{and} \quad u_i'' + v_j'' = d_{ij}, \quad (ij)\epsilon J_B \qquad (2.5)$$

Then, using these variables u_i', v_j', u_i'' *and* v_j'' we define the following 'reduced costs' Δ_{ij}' *and* Δ_{ij}''

$$\left.\begin{array}{l}\Delta_{ij}' = u_i' + v_j' - p_{ij}\\ \Delta_{ij}'' = u_i'' + v_j'' - d_{ij}\end{array}\right\} for\ \ i = 1,2,\dots,m, j = 1,2,\dots,n \qquad (2.6)$$

It is easy to show that the latter may also be expressed as follows

$$\Delta_{ij}(x) = \Delta_{ij}' - Q(x)\Delta_{ij}'', i = 1,2,\dots,m, j = 1,2,\dots,n \qquad (2.7)$$

THEOREM 2.1: (Bajalinov, 2012) Basic feasible solution $x = (x_{ij})$ of LFPT problem is optimal if

$$\Delta_{ij}(x) \geq 0, i = 1,2,\dots,m, j = 1,2,\dots,n \qquad (2.8)$$

Bi-objective Fractional Transportation Problem

Consider the following Bi-objective Fractional Transportation Problems (BFTP):

$$\text{(BFTP)} \quad \text{Maximize} \quad Q_1 = \frac{P_1(x)}{D_1(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^1 x_{ij} + p_0^1}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^1 x_{ij} + d_0^1}$$

$$\text{Maximize} \quad Q_2 = \frac{P_2(x)}{D_2(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^2 x_{ij} + p_0^2}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^2 x_{ij} + d_0^2}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad for\ \ i = 1,2,\dots,m \qquad (3.1)$$

$$\sum_{i=1}^{n} x_{ij} = b_j \quad for\ \ j = 1,2,\dots,n \qquad (3.2)$$

$$x_{ij} \geq 0, \ \ i = 1,2,\dots,m; j = 1,2,\dots,n \qquad (3.3)$$

Here and in what follows we suppose that $D_1(x) > 0$, $D_2(x) > 0, \forall x = (x_{ij}) \in S$, where **S** denotes a feasible set defined by constraints (1.1) to (1.3). Further, we assume that $a_i > 0,\ b_j > 0,\ i = 1,2,\dots,m; j = 1,2,\dots,n$ and total demand equals to total supply, *i.e.* $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

Definition 3.1: (Bajalinov, 2012) A set $X^0 = \{x_{ij}^0, i = 1,2,\dots,m; j = 1,2,\dots,n\}$ is said to be feasible to the problem (BFTP) if X^0 satisfies the conditions (3.1) to (3.3)

Definition 3.2: (Kornbluth, R.E. Steuer , 1981) A feasible solution $X^0$is said to be an efficient solution to the problem (BFTP) if there exists no other feasible X of BTP such that $Q_1(X^0) \geq Q_1(X)$ and $Q_2(X^0) > Q_2(X)$ or $Q_2(X^0) \geq Q_2(X)$ and $Q_1(X^0) > Q_1(X)$. Otherwise, it is called non-efficient solution to the problem (LFTP).

Definition 3.3: (Pandian and Jayalakshmi, m2013) The percentage level of satisfaction of the objective of the transportation problem for the solution U to the transportation problem, $L(Z_t; U)$ is defined as follows:

$$L(Z_t, U) = \begin{cases} \left(\dfrac{Z_t(U)}{Z_t(X_t^0)}\right) \times 100 & \text{if the problem is maximization type} \\ \left(\dfrac{2Z_t(X_t^0) - Z_t(U)}{Z_t(X_t^0)}\right) & \text{if the problem is minimization type} \end{cases}$$

Where $Z_t(U)$ is the objective value at the solution U and $Z_t(X_t^0)$ is the optimal objective value of the transportation problem.

Proposed Method:

We now propose a new algorithm for finding all the solutions to the bi-objective fractional transportation problems (BFTP).The algorithm proceeds as follows:

Step 1: Construct two individual problems of the given BFTP namely, first objective fractional transportation problem (FOFTP) and second objective fractional transportation problem (SOFTP).
Step 2: Obtain an optimal solution to the problems FOFTP and SOFTP using transportation algorithm of fractional programming problems.
Step 3: Start with an optimal solution of FOFTP in the SOFTP as a feasible solution, which is an efficient solution to BTFP.
Step 4: Select the allocated cell (t, r) with the maximum of difference of profit and penalty in the SOFTP. Then, construct a rectangular loop\that starts and ends at the allocated cell (*t*, *r*) and connect some of the unallocated and allocated cells.
Step 5: Add and subtract λ to and from the transition cells of the loop, respectively, in such a way that the rim requirements remain satisfied and assign a sequence of values to λ one by one in such a way that the allocated cell remains non-negative. Then, obtain a feasible solution to SOTFP for each value of λ which is also an efficient / a non-efficient solution to BFTP
Step 6: Check whether the feasible solution to SOFTP obtained from the step 5 is the optimum solution. If not, repeat the Steps 4 and 5 until an optimum solution to SOFTP is found. If so, the process stops and go to the next step.
Step 7: Start with an optimal solution of the SOFTP in the FOFTP as a feasible solution which is an efficient/ non-efficient solution to BFTP.
Step 8: Repeat the steps 4, 5 and 6 for the FOFTP.
Step 9: Combine all obtained solutions (efficient / non-efficient) of BFTP using the optimal solutions of FOTFP and SOFTP. From this, a set of efficient solutions and a set of non- efficient solutions to the BTFP can be obtained.
Step 10: The decision maker selects the preferred optimal solution from the table containing satisfaction level

Numerical Example:

The proposed method for solving a BFTP is illustrated by the following example.

Example: Assume there are two objectives under consideration: The first objective function is the maximization of the ratio of the total delivery speed to total waste along the shipping route and the second objective function is the maximization of ratio of total profit to total cost.The ratio of the total delivery speed to total waste along the shipping route and the second objective function is the maximization of ratio of total profit to total cost are given in the following tables:

(FOFTP):

Destination→ source↓	1	2	3	4	supply
1	4 1	6 3	5 4	2 6	15
2	2 4	5 3	1 6	4 2	25
3	2 5	1 3	4 3	3 2	20
demand	14	18	12	16	

(SOFTP):

Destination→ source↓	1	2	3	4	supply
1	10 15	14 12	8 16	12 8	15
2	8 10	12 6	14 13	8 12	25
3	9 13	6 15	15 12	9 10	20
demand	14	18	12	16	

FOFTP optimal solution is:
$x_{11} = 14\,, x_{12} = 1\,, x_{21} = 17\,, x_{24} = 8\,, x_{33} = 12\,, x_{34} = 8$

SOFTP optimal solution is:
$x_{14} = 15, x_{21} = 7\,, x_{22} = 18\,, x_{31} = 7\,, x_{33} = 12\,, x_{34} = 1$

Now, as in Step 3, we consider the optimal solution of the FOFTP in the SOFTP as a feasible solution in the following table:

Destination→ source↓	1	2	3	4	supply
1	10 15 **14**	14 12 **1**	8 16	12 8	15
2	8 10	12 6 **17**	14 13	8 12 **8**	25
3	9 13	6 15	15 12 **12**	9 10 **8**	20
demand	14	18	12	16	

Thus, $(\frac{251}{136}, \frac{674}{644})$ is the bi-objective value of BFTP for the feasible solution $x_{11} = 14\,, x_{12} = 1\,, x_{21} = 17\,, x_{24} = 8\,, x_{33} = 12\,, x_{34} = 8$

According to Step 4, we construct a rectangular loop (2,4) - (2,2) - (1,2) - (1,4) - (2,4) . By using the Step 5, we have the following reduced table.

Destination→ source↓	1	2	3	4	supply
1	10 15 **14**	14 12 $\mathbf{1-\lambda}$	8 16	12 8 $\boldsymbol{\lambda}$	15
2	8 10	12 6 $\mathbf{17+\lambda}$	14 13	8 12 $\mathbf{8-\lambda}$	25
3	9 13	6 15	15 12 **12**	9 10 **8**	20
demand	14	18	12	16	

Now, the current solution to SOFTP is not the optimum solution. Repetition of Step 4 and 5 results in the following feasible solution which is better than the prior feasible solution of SOFTP.

Thus, by using Steps 4 and 5, we obtain the set of all efficient / non-efficient solution from FOFTP to SOFTP is given below:

Iteration	λ	Solution of BTFP	Bi-objective value
1	{0,1}	$x_{14} = 15, x_{21} = 7 - \lambda, x_{22} = 18,$ $x_{24} = \lambda, x_{31} = 7 + \lambda, x_{33} = 12,$ $x_{34} = 1 - \lambda$	$(\frac{199+\lambda}{245+\lambda}, \frac{704}{543+5\lambda})$
2	{1,2,…,6}	$x_{11} = \lambda, x_{14} = 15 - \lambda, x_{21} = 6 - \lambda,$ $x_{22} = 18, x_{24} = 1 + \lambda, x_{31} = 8,$ $x_{33} = 12$	$(\frac{200+4\lambda}{246-7\lambda}, \frac{704-2\lambda}{548+9\lambda})$
3	{1,2,…,9}	$x_{11} = 6, x_{12} = \lambda, x_{14} = 9 - \lambda,$ $x_{22} = 18 - \lambda, x_{24} = 7 + \lambda, x_{31} = 8,$ $x_{33} = 12$	$(\frac{224+3\lambda}{204-4\lambda}, \frac{692-2\lambda}{602+10\lambda})$
4	{1,2,…,8}	$x_{11} = 6 + \lambda, x_{12} = 9 - \lambda, x_{22} = 9,$ $x_{24} = 16, x_{31} = 8 - \lambda, x_{32} = \lambda,$ $x_{33} = 12$	$(\frac{251-3\lambda}{168-4\lambda}, \frac{674-7\lambda}{692+5\lambda})$
5	{1,2,…,8}	$x_{11} = 14, x_{12} = 1, x_{22} = 9 + \lambda,$ $x_{24} = 16 - \lambda, x_{32} = 8 - \lambda, x_{33} = 12, x_{34} = \lambda$	$(\frac{227+3\lambda}{136}, \frac{618+7\lambda}{732-11\lambda})$

Therefore, the set of all solutions S_1 of the BFTP obtained from FOFTP to SOFTP is

$$S_1 = \{\left(\frac{199}{245},\frac{704}{543}\right),\left(\frac{200}{246},\frac{704}{548}\right),\left(\frac{204}{239},\frac{702}{557}\right),\left(\frac{208}{232},\frac{700}{566}\right),\left(\frac{212}{225},\frac{698}{575}\right),\left(\frac{216}{218},\frac{696}{584}\right),$$

$$\left(\frac{220}{211},\frac{694}{593}\right),\left(\frac{224}{204},\frac{692}{602}\right),\left(\frac{227}{200},\frac{690}{612}\right),\left(\frac{230}{196},\frac{688}{622}\right),\left(\frac{233}{192},\frac{686}{632}\right),\left(\frac{236}{188},\frac{684}{642}\right),$$

$$\left(\frac{239}{184},\frac{682}{652}\right),\left(\frac{242}{180},\frac{680}{662}\right),\left(\frac{245}{176},\frac{678}{672}\right),\left(\frac{248}{172},\frac{676}{682}\right),\left(\frac{251}{168},\frac{674}{692}\right),\left(\frac{248}{164},\frac{667}{697}\right),$$

$$\left(\frac{245}{160},\frac{660}{702}\right),\left(\frac{242}{156},\frac{653}{707}\right),\left(\frac{239}{152},\frac{646}{712}\right),\left(\frac{236}{148},\frac{639}{717}\right),\left(\frac{233}{144},\frac{632}{722}\right),\left(\frac{230}{140},\frac{625}{727}\right),$$

$$\left(\frac{227}{136},\frac{618}{732}\right),\left(\frac{230}{136},\frac{625}{721}\right),\left(\frac{233}{136},\frac{632}{710}\right),\left(\frac{236}{136},\frac{639}{699}\right),\left(\frac{239}{136},\frac{646}{688}\right),\left(\frac{242}{136},\frac{653}{677}\right),$$

$$\left(\frac{245}{136},\frac{660}{666}\right),\left(\frac{248}{136},\frac{667}{655}\right),\left(\frac{251}{136},\frac{674}{644}\right)\}$$

Similarly, by using Steps 7 and 8, we obtain the set of all solutions S_2 from SOFTP to FOFTP is given below:

Iteration	λ	Solution of BTFP	Bi-objective value
1	{0,1}	$x_{11} = 14, x_{12} = 1 - \lambda, x_{14} = \lambda,$ $x_{22} = 17 + \lambda, x_{24} = 8 - \lambda, x_{33} = 12,$ $x_{34} = 8$	$(\frac{251-3\lambda}{136+4\lambda}, \frac{674+2\lambda}{644-10\lambda})$
2	{1,2,…,7}	$x_{11} = 14, x_{14} = 1, x_{22} = 18,$ $x_{23} = \lambda, x_{24} = 7 - \lambda, x_{33} = 12 - \lambda,$ $x_{34} = 8 + \lambda$	$(\frac{248-4\lambda}{140+3\lambda}, \frac{676}{634-\lambda})$
3	{1,2,…,14}	$x_{11} = 14 - \lambda, x_{14} = 1 + \lambda, x_{22} = 18,$ $x_{23} = 7, x_{31} = \lambda, x_{33} = 5,$ $x_{34} = 15 - \lambda$	$(\frac{220-3\lambda}{161+8\lambda}, \frac{676+2\lambda}{627-4\lambda})$
4	{1,2,…,7}	$x_{14} = 15, x_{21} = \lambda, x_{22} = 18,$ $x_{23} = 7 - \lambda, x_{31} = 14 - \lambda,$ $x_{33} = 5 + \lambda, x_{34} = 1$	$(\frac{178+3\lambda}{273-4\lambda}, \frac{704}{571-4\lambda})$

Therefore, the set of all solutions S_2 of the BFTP obtained from SOFTP to FOFTP is

$$S_2=\{\left(\frac{251}{136},\frac{674}{644}\right),\left(\frac{248}{140},\frac{676}{634}\right),\left(\frac{244}{143},\frac{676}{633}\right),\left(\frac{240}{146},\frac{676}{632}\right),\left(\frac{236}{149},\frac{676}{631}\right),\left(\frac{232}{152},\frac{676}{630}\right),$$

$$\left(\frac{228}{155},\frac{676}{629}\right),\left(\frac{224}{158},\frac{676}{628}\right),\left(\frac{220}{161},\frac{676}{627}\right),\left(\frac{217}{169},\frac{678}{623}\right),\left(\frac{214}{177},\frac{680}{619}\right),\left(\frac{211}{185},\frac{682}{615}\right),$$

$$\left(\frac{208}{193},\frac{684}{611}\right),\left(\frac{205}{201},\frac{686}{607}\right),\left(\frac{202}{209},\frac{688}{603}\right),\left(\frac{199}{217},\frac{690}{599}\right),\left(\frac{196}{225},\frac{692}{595}\right),\left(\frac{193}{233},\frac{694}{591}\right),$$

$$\left(\frac{190}{241},\frac{696}{587}\right),\left(\frac{187}{249},\frac{698}{583}\right),\left(\frac{184}{257},\frac{700}{579}\right),\left(\frac{181}{265},\frac{702}{575}\right),\left(\frac{178}{273},\frac{704}{571}\right),\left(\frac{181}{269},\frac{704}{567}\right),$$

$$\left(\frac{184}{265},\frac{704}{563}\right),\left(\frac{187}{261},\frac{704}{559}\right),\left(\frac{190}{257},\frac{704}{555}\right),\left(\frac{193}{253},\frac{704}{551}\right),\left(\frac{196}{249},\frac{704}{547}\right),\left(\frac{199}{245},\frac{704}{543}\right)\}$$

Now the following table shows the set of all solutions S of the BFTP obtained from FOFTP to SOFTP and from SOFTP to FOFTP and the satisfaction level of objectives of the problem at each efficient solution:

	Bi-objective value of BTP	Level	Satisfaction
		Objective of FOTP	Objective of SOTP
1	$\left(\frac{199}{245},\frac{704}{543}\right)\simeq(0.812,1.296)$	44.01	100
2	$\left(\frac{200}{246},\frac{704}{548}\right)\simeq(0.813,1.284)$	44.05	99.08
3	$\left(\frac{204}{239},\frac{702}{557}\right)\simeq(0.853,1.26)$	46.24	97.20
4	$\left(\frac{208}{232},\frac{700}{566}\right)\simeq(0.896,1.236)$	48.57	95.39
5	$\left(\frac{212}{225},\frac{698}{575}\right)\simeq(0.942,1.213)$	51.05	93.62
6	$\left(\frac{216}{218},\frac{696}{584}\right)\simeq(0.991,1.191)$	53.68	91.92
7	$\left(\frac{220}{211},\frac{694}{593}\right)\simeq(1.042,1.170)$	56.49	90.26
8	$\left(\frac{224}{204},\frac{692}{602}\right)\simeq(1.098,1.149)$	59.49	88.66
9	$\left(\frac{227}{200},\frac{690}{612}\right)\simeq(1.135,1.127)$	61.49	86.96
10	$\left(\frac{230}{196},\frac{688}{622}\right)\simeq(1.173,1.106)$	63.58	85.31
11	$\left(\frac{233}{192},\frac{686}{632}\right)\simeq(1.213,1.085)$	65.75	83.72
12	$\left(\frac{236}{188},\frac{684}{642}\right)\simeq(1.255,1.065)$	68.01	82.17
13	$\left(\frac{251}{136},\frac{674}{644}\right)\simeq(1.845,1.046)$	100	80.72
14	$\left(\frac{248}{140},\frac{676}{634}\right)\simeq(1.771,1.066)$	95.98	82.24
15	$\left(\frac{244}{143},\frac{676}{633}\right)\simeq(1.706,1.067)$	92.45	82.37
16	$\left(\frac{240}{146},\frac{676}{632}\right)\simeq(1.643,1.069)$	89.06	82.50
17	$\left(\frac{236}{149},\frac{676}{631}\right)\simeq(1.583,1.071)$	85.82	82.63
18	$\left(\frac{232}{152},\frac{676}{630}\right)\simeq(1.526,1.073)$	82.70	82.76
19	$\left(\frac{228}{155},\frac{676}{629}\right)\simeq(1.471,1.074)$	79.70	82.89
20	$\left(\frac{224}{158},\frac{676}{628}\right)\simeq(1.417,1.076)$	76.81	83.02
21	$\left(\frac{220}{161},\frac{676}{627}\right)\simeq(1.366,1.078)$	74.03	83.15
22	$\left(\frac{217}{169},\frac{678}{623}\right)\simeq(1.284,1.088)$	69.57	83.93
23	$\left(\frac{214}{177},\frac{680}{619}\right)\simeq(1.209,1.098)$	65.50	84.73
24	$\left(\frac{211}{185},\frac{682}{615}\right)\simeq(1.14,1.108)$	61.79	85.53

25	$\left(\frac{208}{193},\frac{684}{611}\right) \simeq(1.077,1.119)$	58.39	86.34
26	$\left(\frac{205}{201},\frac{686}{607}\right) \simeq(1.019,1.13)$	55.26	87.16
27	$\left(\frac{202}{209},\frac{688}{603}\right) \simeq(0.966,1.141)$	52.36	88.00
28	$\left(\frac{199}{217},\frac{690}{599}\right) \simeq(0.917,1.151)$	49.68	88.84
29	$\left(\frac{196}{225},\frac{692}{595}\right) \simeq(0.871,1.163)$	47.19	89.70
30	$\left(\frac{193}{233},\frac{694}{591}\right) \simeq(0.828,1.174)$	44.88	90.57
31	$\left(\frac{190}{241},\frac{696}{587}\right) \simeq(0.788,1.185)$	42.71	91.45
32	$\left(\frac{196}{249},\frac{704}{547}\right) \simeq(0.787,1.287)$	42.65	99.26

The above satisfaction level table is very much useful for the decision makers to select the appropriate efficient solutions to bi-objective transportation problems according to their level of satisfaction of objectives

Conclusion:

One of the important economic aspects for multi-objective transportation is the determination of efficient distributions for a given commodity between source and destination. In this paper, the proposed method determines the set of efficient solutions for bi-objective fractional transportation problems. In proposed method, unlike other methods for multi-objective problems without using auxiliary variables, and from one solution, we obtain the next-useful solution. This method enables the decision makers to select an appropriate solution and can determine the preferred solution from efficient solution using it. Often, in multi-objective transportation problem, the coefficients of the objective functions and supply and demand are fuzzy data that these values are determined by the decision maker. We suggest that the decision maker use this method to solve fuzzy multi-objective transport problems.

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