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Dynamical Behavior of Hepatitis B Fractional-order Model with Modeling and Simulation

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Abstract

We presented a nonlinear time-fractional model of Hepatitis B in order to understand the outbreaks of this epidemic disease. The fractional parameter is used to develop the system of complex nonlinear differential equations by using Caputo sense with fractional order derivative. We investigated the qualitative analysis of the fractional-order model and also the stability of the model was checked through the analysis. Hepatitis B is a highly contagious disease that can spread in a population depending on the number of susceptible people or patients with chronic disease and also depending on their dynamics in the community. The solution of the classical, as well as the time-fractional model, was procured by using LADM. Finally, numerical simulations are also established to investigate the influence of the system parameter on the spread of the disease.

Keywords: Caputo fractional; Hepatitis B; Stability; Modeling and Simulation.

Introduction

Modeling is the human interest including representing, manipulating, and communicating with the real-world's daily life objects. As anyone can effortlessly comprehend, there are various ways to take a look at an item or, equivalently, there are numerous unique observers for the same object. Every observer has 'one kind of perspective' from the same item, i.e. 'there is no omniscient observer with unique get right of entry to the reality'. Each distinct observer collects facts and generates a hypothesis that is constant with the records. This logical procedure is called 'abduction'. Abduction is not always infallible, although; with admire to an unknown systematic, we are all blind.

A system is a set of interrelated objects. For instance, a biological system may be a set of various cell cubicles (e.g. cellular sorts) specialized for a particular biological feature (e.g. white and red blood cells have very distinctive commitments). An item is as some elemental units upon which a statement can be made, however, its inner shape is unknown or does not exist at present. The selection of the basic unit defines the illustration scale of the machine. A model is an outline of a device in terms of constitutive items and the communication between them, which description is generally decodable or interpretable by people (Karplus 1983).

Some epidemics worldwide have led to catastrophic effects for human populations. In the past, the Black dying and Cholera were epidemics, spread across Europe over large distances (Murray 2002). Expertise approximately the unfold and severity of epidemic diseases is important for the human population to stop the most critical damages. The current outbreak of Ebola in West Africa in March 2014 (WHO 2014).

Epidemiological study plays an important role in understanding the impact of infectious disease in a society. In mathematical modeling, we investigate models by model building, performing estimation of the parameters, checking the sensitivity of models by varying parameters and computing their numerical simulations. This kind of research helps to understand the ratio of disease spread in the population and to control its parameters. These types of disease models are often called infectious diseases (i.e. the disease, transferred from one person to another) (Ashraf and Ahmad, 2019; Ahmad et al., 2018).

Hepatitis is a disease characterized by using inflammation of an injury to the liver (Alharbi, et al., 2017). Hepatitis has many reasons, including the misuse of alcohol and pills, but viruses are the most common cause. Hepatitis B is a serious health mission (Shakeri, et al., 2018). More than one billion people across the world had been inflamed by the hepatitis B virus (HBV) and over three hundred million people are carriers of the virus (White and Fenner, 1994; Platkov et al., 2001; Cariappa et al., 2004; Fernandez et al., 2006; Onuzulike and Ogueri, 2007). Fractional calculus (FC) is the extension of daily calculus with more than three hundred years of records. Fractional calculus initiated with the useful resource of Leibniz and L`health facility because of a correspondence, which lasted numerous months in 1695. In that year, Leibniz wrote a letter to L'health facility, raising the subsequent queries. The problem raised with the aid of Leibniz

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for a fractional spinoff (semi-spinoff, to be extra unique) modified into an ongoing subject count in a long term to return (Anselmi et al., 2005; Arsuaga et al., 2002).

The Laplace transforms coupling with Adomian polynomials is numerical a technique, which powerfully works for a system of deterministic as well as stochastic differential equations. In addition, it is observed that LADM is more powerful than standard ADM method (Ashraf et al., 2018; Ahmad et al., 2018). Laplace remodelling technique is a beneficial tool in various types of organic science. It is also applied in implemented arithmetic and in engineering and many others. The combination of ADM with Laplace rework technique is a sturdy method, which is likewise referred to as Laplace Adomain decomposition method. Laplace remodel helps us to trade a differential equation to an algebraic equation and the nonlinear terms are a breakdown in the shape of Adomain polynomials.

Material and Method

We begin by introducing the existing model (Zou et al., 2009).

$$\frac{dS}{dt} = \mu w\,(1 - vC) + \Psi V - (\mu_0 + \beta I + \varepsilon\beta C + \gamma_3)S \quad (1)$$

$$\frac{dL}{dt} = (\beta I + \varepsilon\beta C)\,S - (\sigma + \mu_0)\,L \quad (2)$$

$$\frac{dI}{dt} = \sigma L - (\mu_0 + \gamma_1)I \quad (3)$$

$$\frac{dC}{dt} = \mu v w C + q\,\gamma_1 I - (\mu_0 + \mu_1 + \gamma_2)C \quad (4)$$

$$\frac{dR}{dt} = (1 - q)\,\gamma_1 I + \gamma_2 C - \mu_0 R \quad (5)$$

$$\frac{dV}{dt} = \mu\,(1 - w) + \gamma_3\,S - (\mu_0 + \Psi)\,V \quad (6)$$

Susceptible, latent, acutely infected, chronic carriers, recovered, and vaccinated individuals at time t are S, L, I, C, R, V respectively. Here μ, μ_0, μ_1, ω represents the birth, natural mortality, HBV-related mortality rate, and proportion of births without vaccination. $(1-\omega)$ represents the proportion of vaccinated births, (v) represents the proportion of vertically infected births, (Ψ) is waning vaccine-induced immunity rate, and (σ) represents the latent state to acute state moving rate. (β) is the transmission coefficient, (γ_1) represents the acute to other compartments moving rate, and (q) represents the carrier state. $(q\gamma_1)$, $((1-q)\,\gamma_1)$, and (γ_2) represent acute to the carrier, acute to recovered class, and carrier to immune moving rates, respectively. The vaccination rate of the susceptible individuals (γ_3), reduced transmission rate relative to acute infection by carriers (ε).

Modified Model

- Chronic carriers are preserved at the rate α. (WHO 2001).
- Newborns of carrier mothers infected at birth (Mehmood 2011).
- Treated recovered individuals (O'leary et al., 2010; Abu and Onalo 2017).

$$\frac{dS}{dt} = \mu w\,(1 - vC) + \Psi V - (\mu_0 + \beta I + \varepsilon\beta C + \gamma_3)S \quad (7)$$

$$\frac{dL}{dt} = \mu v w C + (\beta I + \varepsilon\beta C)\,S - (\sigma + \mu_0)\,L \quad (8)$$

$$\frac{dI}{dt} = \sigma L - (\mu_0 + \gamma_1)I \quad (9)$$

$$\frac{dC}{dt} = q\,\gamma_1 I - (\mu_0 + \mu_1 + \gamma_2)C \quad (10)$$

$$\frac{dV}{dt} = \mu\,(1 - w) + \gamma_3\,S - (\mu_0 + \Psi)\,V \quad (11)$$

$$\frac{dR}{dt} = (1 - q)\gamma_1 I + (\gamma_2 + \alpha)C - \mu_0 R \quad (12)$$

with given initial conditions $S(0) \geq 0, L(0) \geq 0, I(0) \geq 0, C(0) \geq 0, V(0) \geq 0, R(0)\,0.$

where $S(t) + L(t) + I(t) + C(t) + R(t) + V(t) = 1$

Fractional Order Model

The fractional system of differential equations (FDEs) by using Caputo Fractional Derivative is as follows:

$$D_t^{\varphi_1} S(t) = \mu w\,(1 - vC) + \Psi V - (\mu_0 + \beta I + \varepsilon\beta C + \gamma_3)S \quad (13)$$

$$D_t^{\varphi_2} L(t) = \mu v w C + (\beta I + \varepsilon\beta C)\,S - (\sigma + \mu_0)\,L \quad (14)$$

$$D_t^{\varphi_3} I(t) = \sigma L - (\mu_0 + \gamma_1)I \quad (15)$$

$$D_t^{\varphi_4} C(t) = q\,\gamma_1 I - (\mu_0 + \mu_1 + \gamma_2)C \quad (16)$$

$$D_t^{\varphi_5} V(t) = \mu\,(1 - w) + \gamma_3\,S - (\mu_0 + \Psi)\,V \quad (17)$$

$$D_t^{\varphi_6} R(t) = (1 - q)\gamma_1 I + (\gamma_2 + \alpha)C - \mu_0 R \quad (18)$$

Subject to the conditions

$$S(0) = N_1 = 0.7\,, L(0) = N_2 = 0.05\,, I(0) = N_3 = 0.05\,, C(0) = N_4 = 0.08\,, V(0) = N_5 = 0.06\,, R(0) = N_6 = 0.12$$

Qualitative Analysis

Consider left hand side of the above system equal to zero for the equilibrium point

$$D^{\varphi_1}_t S(t) = D^{\varphi_2}_t L(t) = D^{\varphi_3}_t I(t) = D^{\varphi_4}_t C(t) = D^{\varphi_5}_t V(t) = D^{\varphi_6}_t R(t) = 0$$

we get (S = 1.0207 , L = 0 , I = 0 , C = 0 , V = 1.19014 , R = 0) So,

Disease Free point is

$$(S_0, L_0, I_0, C_0, V_0, R_0) = (1.0207\,, 0\,, 0\,, 0\,, 1.19014\,, 0)$$

After simplification in form of parameter,

We get the endemic point

$$(S_E, L_E, I_E\,, C_E\,, V_E\,, R_E\,) = \left(\frac{\mu\Psi + \mu\mu_0 w}{\mu_\circ(\gamma_3 + \Psi + \mu_0)}, 0, 0, 0, \frac{\gamma_3\mu + \mu\mu_0 - \mu\mu_0 w}{\mu_\circ(\gamma_3 + \Psi + \mu_0)}, 0\right)$$

Theorem-1: E_0 is locally stable if $R_e(\lambda) < 0$, otherwise unstable.

Proof: Consider a Jacobian matrix

$$J = \begin{bmatrix} -(\mu_0 + \beta\, I + \varepsilon\beta C + \gamma_3) & 0 & -\beta\, S & -\mu w v - \varepsilon\beta S & \Psi & 0 \\ \beta I + \varepsilon\beta C & -(\sigma + \mu_0) & \beta\, S & \mu w v + \varepsilon\beta S & 0 & 0 \\ 0 & \sigma & -(\mu_0 + \gamma_1) & 0 & 0 & 0 \\ 0 & 0 & q\gamma_1 & -(\mu_0 + \mu_1 + \gamma_2 + \alpha) & 0 & 0 \\ \gamma_3 & 0 & 0 & 0 & -(\mu_0 + \Psi) & 0 \\ 0 & 0 & (1-q)\gamma_1 & \gamma_2 + \alpha & 0 & -\mu_0 \end{bmatrix}$$

$$J_{E_0} = \begin{bmatrix} -(\mu_0 + \gamma_3) & 0 & -\beta\, S & -\mu w v - \varepsilon\beta S & \Psi \\ 0 & -(\sigma + \mu_0) & \beta\, S & \mu w v + \varepsilon\beta S & 0 \\ 0 & \sigma & -(\mu_0 + \gamma_1) & 0 & 0 \\ 0 & 0 & q\gamma_1 & -(\mu_0 + \mu_1 + \gamma_2 + \alpha) & 0 \\ \gamma_3 & 0 & 0 & 0 & -(\mu_0 + \Psi) \end{bmatrix}$$

$$|J_0 - \lambda\, I| = 0$$

$$\begin{vmatrix} -(\mu_0 + \gamma_3 + \lambda) & 0 & -\beta\, S & -\mu w v - \varepsilon\beta S & \Psi \\ 0 & -(\sigma + \mu_0 + \lambda) & \beta\, S & \mu w v + \varepsilon\beta S & 0 \\ 0 & \sigma & -(\mu_\circ + \gamma_1 + \lambda) & 0 & 0 \\ 0 & 0 & q\gamma_1 & -(\mu_0 + \mu_1 + \gamma_2 + \alpha + \lambda) & 0 \\ \gamma_3 & 0 & 0 & 0 & -(\mu_0 + \Psi + \lambda) \end{vmatrix} = 0$$

$$\lambda_1 = -0.2166 < 0, \lambda_2 = -0.0166 < 0, \lambda_3 = -0.0744 < 0, \lambda_4 = -7.5395 < 0, \lambda_5 = -2.6808 < 0.$$

All the eigenvalues of real parts are negative. Therefore, the given system is stable.

The fractional-order extension of this model has been studied. They explicit realistic biphasic flip-down movement of the infection but at a slower pace. The fresh gadget of the differential equation is represented with the fractional scheme of differential equations (FDEs) in the system (13-18). By using the definition of the Laplace transform, we have:

$$\mathcal{L}\{D_t^{\varphi_1} S(t)\} = \mathcal{L}\{\mu w\,(1 - vC) + \Psi V - (\mu_0 + \beta I + \varepsilon\beta C + \gamma_3)S\} \quad (19)$$

$$\mathcal{L}\{D_t^{\varphi_2} L(t)\} = \{\mu v w C + (\beta I + \varepsilon\beta C)S - (\sigma + \mu_0)L\} \quad (20)$$

$$\mathcal{L}\{D_t^{\varphi_3} I(t)\} = \mathcal{L}\{\sigma L - (\mu_0 + \gamma_1)I\} \quad (21)$$

$$\mathcal{L}\{D_t^{\varphi_4} C(t)\} = \mathcal{L}\{q\,\gamma_1 I - (\mu_0 + \mu_1 + \gamma_2)C\} \quad (22)$$

$$\mathcal{L}\{D_t^{\varphi_5} V(t)\} = \mathcal{L}\{\mu\,(1 - w) + \gamma_3\, S - (\mu_0 + \Psi)\, V\} \quad (23)$$

$$\mathcal{L}\{D_t^{\varphi_6} R(t)\} = \mathcal{L}\{(1 - q)\gamma_1 I + (\gamma_2 + \alpha)C - \mu_0 R\} \quad (24)$$

Subject to the following initial conditions

$S(0) = N_1 = 0.7, L(0) = N_2 = 0.05, I(0) = N_3 = 0.05, C(0) = N_4 = 0.08, V(0) = N_5 = 0.06, R(0) = N_6 = 0.12$ (25)

By putting

$$S_0 = N_1 \ , \ L_0 = N_2 \ , \ I_0 = N_3 \ , \ C_0 = N_4 \ , \ V_0 = N_5 \ , \ R_0 = N_6$$

We get,

$$\mathcal{L}\{S\} = \frac{N1}{s} + \frac{\mu w}{S^{\varphi_1+1}} - \frac{\mu w v}{S^{\varphi_1}}\mathcal{L}(C) - \frac{\varepsilon\beta}{S^{\varphi_1}}\mathcal{L}(CS) - \frac{(\mu_0+\gamma 3)}{S^{\varphi_1}}\mathcal{L}(S) - \frac{\beta}{S^{\varphi_1}}\mathcal{L}(IS)$$

$$\mathcal{L}\{L\} = \frac{N2}{s} + \frac{\mu v w}{S^{\varphi_2}}\mathcal{L}(C) + \frac{\varepsilon\beta}{S^{\varphi_2}}\mathcal{L}(CS) - \frac{(\sigma+\mu_0)}{S^{\varphi_2}}\mathcal{L}(L) + \frac{\beta}{S^{\varphi_2}}\mathcal{L}(IS)$$

$$\mathcal{L}\{I\} = \frac{N_3}{s} - \frac{(\mu_0 + \gamma 1)}{S^{\varphi_3}}\mathcal{L}(I) + \frac{\sigma}{S^{\varphi_3}}\mathcal{L}(L)$$

$$\mathcal{L}\{C\} = \frac{N_4}{s} + \frac{q\gamma 1}{S^{\varphi_4}}\mathcal{L}(I) - \frac{(\mu_0+\mu_1+\gamma_2+\alpha)}{S^{\varphi_4}}\mathcal{L}(C) \quad (26)$$

$$\mathcal{L}\{V\} = \frac{N_5}{s} + \frac{\mu(1-w)}{S^{\varphi_5+1}} + \frac{\gamma 3}{S^{\varphi_5}}\mathcal{L}(S) - \frac{(\mu_0+\Psi)}{S^{\varphi_5}}\mathcal{L}(V)$$

$$\mathcal{L}\{R\} = \frac{N_6}{s} + \frac{(1-q)\gamma 1}{S^{\varphi_6}}\mathcal{L}(I) + \frac{(\gamma 2+\alpha)}{S^{\varphi_6}}\mathcal{L}(C) - \frac{\mu_0}{S^{\varphi_6}}\mathcal{L}(R)$$

Suppose that

$$S = \sum_{k=0}^{\infty} S_k \quad , \quad L = \sum_{k=0}^{\infty} L_k \quad , \quad R = \sum_{k=0}^{\infty} R_k$$

$$I = \sum_{k=0}^{\infty} I_k \quad , \quad C = \sum_{k=0}^{\infty} C_k \quad , \quad V = \sum_{k=0}^{\infty} V_k$$

The nonlinearity can be written as

$$SI = \sum_{k=0}^{\infty} A_k \quad , \ SC = \sum_{k=0}^{\infty} D_k$$

$$\mathcal{L}\{SK+1\} = -\frac{\mu w v}{S^{\varphi_1}}\mathcal{L}\{Ck\} - \frac{\varepsilon\beta}{S^{\varphi_1}}\mathcal{L}(Dk) - \frac{(\mu_0+\gamma 3)}{S^{\varphi_1}}\mathcal{L}\{Sk\} - \frac{\beta}{S^{\varphi_1}}\mathcal{L}\{Ak\}$$

$$\mathcal{L}\{Lk+1\} = \frac{\mu v w}{S^{\varphi_2}}\mathcal{L}(Ck) + \frac{\varepsilon\beta}{S^{\varphi_2}}\mathcal{L}\{Dk\} - \frac{(\sigma+\mu_0)}{S^{\varphi_2}}\mathcal{L}\{Lk\} + \frac{\beta}{S^{\varphi_2}}\mathcal{L}\{Ak\}$$

$$\mathcal{L}\{Ik+1\} = -\frac{(\mu_0+\gamma 1)}{S^{\varphi_3}}\mathcal{L}\{Ik\} + \frac{\sigma}{S^{\varphi_3}}\mathcal{L}\{Lk\}$$

$$\mathcal{L}\{Ck+1\} = \frac{q\gamma_1}{S^{\varphi_4}}\mathcal{L}\{Ik\} - \frac{(\mu_0+\mu_1+\gamma_2+\alpha)}{S^{\varphi_4}}\mathcal{L}\{Ck\}$$

$$\mathcal{L}\{Vk+1\} = \frac{\gamma_3}{S^{\varphi_5}}\mathcal{L}(Sk) - \frac{(\mu_0+\Psi)}{S^{\varphi_5}}\mathcal{L}(Vk)$$

$$\mathcal{L}\{Rk+1\} = \frac{(1-q)\gamma_1}{S^{\varphi_6}}\mathcal{L}(Ik) + \frac{(\gamma_2+\alpha)}{S^{\varphi_6}}\mathcal{L}(Ck) - \frac{\mu_0}{S^{\varphi_6}}\mathcal{L}(Rk)$$

$$S(t) = 0.7\frac{t^{\varphi_1}}{\varphi_1!} - 0.0123\frac{t^{\varphi_1}}{\varphi_1!} + 0.02167\frac{t^{\varphi_1}}{\varphi_1!} - 0.0658\frac{t^{\varphi_3}}{\varphi_3!} - 0.165\frac{t^{\varphi_4}}{\varphi_4!} + 0.0274\frac{t^{\varphi_1+\varphi_4}}{(\varphi_1+\varphi_4)!} + 0.00115\frac{t^{2\varphi_1}}{2\varphi_1!} + 0.0405\frac{t^{\varphi_1+\varphi_3}}{(\varphi_1+\varphi_3)!}\ldots.$$

$$L(t) = 0.05 + 0.065\frac{t^{\varphi_2+\varphi_3}}{(\varphi_2+\varphi_3)!} - 0.2593\frac{t^{\varphi_2}}{\varphi_2!} - 5.8439\frac{t^{\varphi_1+\varphi_2}}{(\varphi_1+\varphi_2)!} + 0.0165\frac{t^{\varphi_2+\varphi_4}}{(\varphi_2+\varphi_4)!} - 1.558\frac{t^{2\varphi_1}}{2\varphi_1!} + \cdots$$

$$I(t) = 0.05 + 0.099\frac{t^{\varphi_3}}{\varphi_3!} - 1.554\frac{t^{\varphi_2+\varphi_3}}{(\varphi_2+\varphi_3)!} - 0.397\frac{t^{2\varphi_3}}{2\varphi_3!} + \cdots$$

$$C(t) = 0.08 + 0.1561\frac{t^{\varphi_4}}{\varphi_4!} + 0.3504\frac{t^{\varphi_3+\varphi_4}}{(\varphi_3+\varphi_4)!} - 0.0408\frac{t^{2\varphi_4}}{2\varphi_4!} - 5.50116\frac{t^{\varphi_2+\varphi_3+\varphi_4}}{(\varphi_2+\varphi_3+\varphi_4)!} - 1.4054\frac{t^{2\varphi_3+\varphi_4}}{(2\varphi_3+\varphi_4)!} - 0.09166\frac{t^{\varphi_3+2\varphi_4}}{(\varphi_3+2\varphi_4)!} + 0.01067\frac{t^{3\varphi_4}}{3\varphi_4!} + \cdots$$

$V(t) = V_0 + V_1 + V_2 + \ldots$

$$V(t) = 0.06 + 0.0997\frac{t^{\varphi_5}}{\varphi_5!} - 0.0123\frac{t^{\varphi_1+\varphi_5}}{(\varphi_1+\varphi_5)!} - 0.01162\frac{t^{2\varphi_5}}{2\varphi_5!} - 0.00049895\frac{t^{3\varphi_5}}{3\varphi_5!} + \ldots$$

$$R(t) = 0.12 + 0.023\frac{t^{\varphi_6}}{\varphi_6!} + 0.045\frac{t^{\varphi_3+\varphi_6}}{(\varphi_3+\varphi_6)!} + 3.9 * 10^{-3}\frac{t^{\varphi_4+\varphi_6}}{(\varphi_4+\varphi_6)!} - 3.18 * 10^{-4}\frac{t^{2\varphi_6}}{2\varphi_6!} + \cdots$$

Which is the required series solution.

Results and Discussions:

The mathematical analysis of hepatitis B model has been present to observe the effects of parameters. Numerical results

for different fractions values can be seen in table 2 to 7. Fig 1 clearly shows that our results on fractional parameters are given within the domain, but the results failed in the classical order (fig 2-4). The infected individual's acute or chronic disease starts decreasing by decreasing the values of fractional parameters, the similar behavior can be observed in fig (5-6) for vaccinated individuals as well as for recovered populations.

Parameter values used in the mathematical model are shown in table 1.

Table 1: Values of physical parameters

Parameters	**Value**	**Parameters**	**Value**
V	0.11	μ_1	0.02
Ψ	0.1	γ_3	0.1
Σ	6 per year	A	0.2
B	0.95	W	0
γ_1	4 per year	S(0)	0.7
Q	0.885	L(0)	0.05
γ_2	0.025	I(0)	0.05
ε	0.16	C(0)	0.08
μ	0.0367	R(0)	0.12
μ_0	0.0166	V(0)	0.06

Table 2: Susceptible individuals S (t) at different fractional values

T	$\varphi_i = 1$	$\varphi_i = 0.95$	$\varphi_i = 0.9$
0	0.7	0.7	0.7
0.2	0.5892	0.6265	0.6189
0.4	0.4785	0.5580	0.5486
0.6	0.3678	0.4913	0.4819
0.8	0.2572	0.4257	0.4175
1	0.1465	0.3610	0.3547

Table 3: Latent individuals L (t) at different fractional values

T	$\varphi_i = 1$	$\varphi_i = 0.95$	$\varphi_i = 0.9$
0	0.05	0.05	0.05
0.01	0.0467	0.0455	0.0439
0.02	0.0424	0.0399	0.0366
0.03	0.0373	0.0332	0.0277
0.04	0.0312	0.0255	0.0175
0.05	0.0243	0.0165	0.0059

Table 4: Acutely infected individuals I (t) at different fractional values

T	$\varphi_i = 1$	$\varphi_i = 0.95$	$\varphi_i = 0.9$
0	0.05	0.05	0.05
0.03	0.0522	0.0523	0.0533
0.06	0.0523	0.0515	0.0537
0.10	0.0501	0.0479	0.0520
0.13	0.0458	0.0417	0.0484
0.16	0.0394	0.0329	0.0431

Table 5: Chronic carriers C (t) at different fractional values

T	$\varphi_i = 1$	$\varphi_i = 0.95$	$\varphi_i = 0.9$
0	0.08	0.08	0.08
0.2	0.1174	0.1225	0.1283
0.4	0.1672	0.1764	0.1866
0.6	0.2294	0.2422	0.2561
0.8	0.3039	0.3196	0.3363
1	0.3909	0.4087	0.4269

Table 6: Vaccinated individuals V (t) at different fractional values

T	$\varphi_i = 1$	$\varphi_i = 0.95$	$\varphi_i = 0.9$
0	0.06	0.06	0.06
0.2	0.0794	0.2799	0.0835
0.4	0.0979	0.4837	0.1026
0.6	0.1154	0.6813	0.1196
0.8	0.1320	0.8745	0.1351
1	0.1475	1.0643	0.1492

Table 7: Recovered individuals R (t) at different fractional values

T	$\varphi_i = 1$	$\varphi_i = 0.95$	$\varphi_i = 0.9$
0	0.12	0.12	0.12
0.2	0.1256	0.1263	0.1272
0.4	0.1331	0.1344	0.1360
0.6	0.1425	0.1444	0.1466
0.8	0.1539	0.1562	0.1589
1	0.1673	0.1699	0.1728

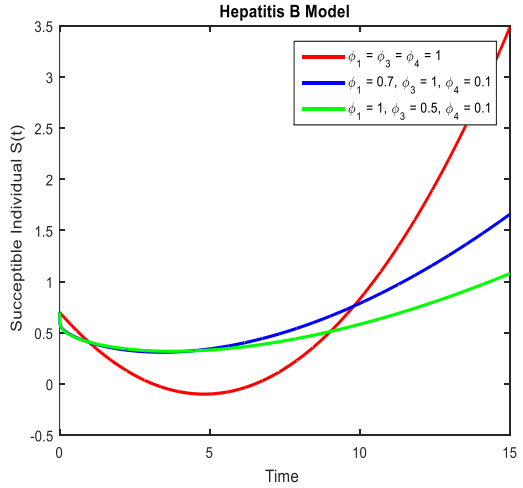

Fig 1: Numerical solution of susceptible individuals S (t) at different values of φ

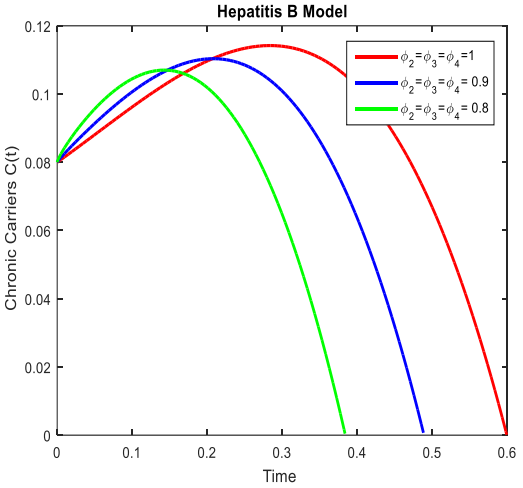

Fig 4: Numerical solution of chronic carriers C (t) at different values of φ

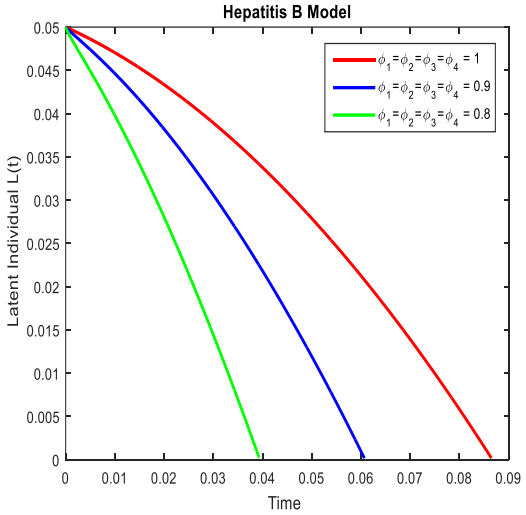

Fig 2: Numerical solution of latent individuals L (t) at different values of φ

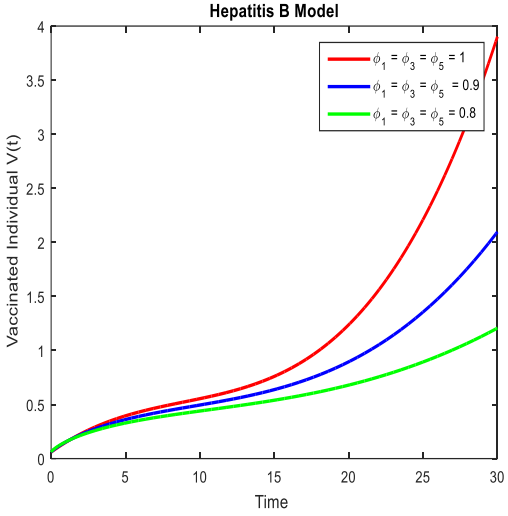

Fig 5: Numerical solution of vaccinated individuals V (t) at different values of φ

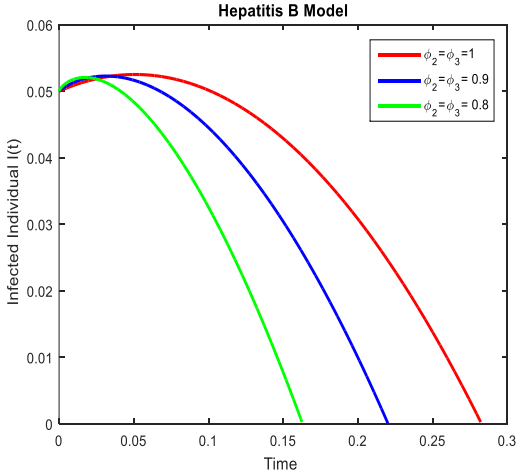

Fig 3: Numerical solution of acutely infected individuals I (t) at different values of φ

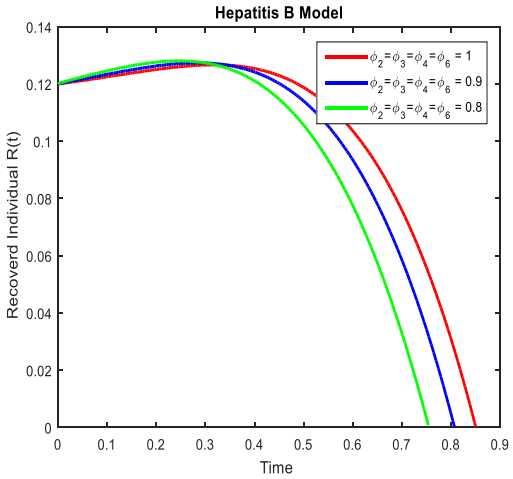

Fig 6: Numerical solution of recovered individuals R (t) at different values of φ

Conclusion

Hepatitis B is one of the serious diseases in the community, which affects the healthy life. If people use effective parameters of controlling and prevention, they will develop the disease. It also affects human life in the future (Al-Eisa, 2017). Various techniques and analyses are used to analyze and evaluate the sensitivity and measure the effect of parameters to control the disease in society. For this purpose, in this study, we developed the scheme fractional order mathematical model for Hepatitis B by using the Caputo sense. The mathematical model characterizes the dynamical system during the disease as a set of non-linear coupled ordinary differential equations. Qualitative and stability analysis has been made for convergence the scheme of epidemic fractional Hepatitis B model. The numerical simulation in figures and tables show the impact, spread, and control during the time. The effect of fractional parameter can be analyzed through Tables and graphs on our obtained solutions. It is praiseworthy to perceive that fractional derivatives show important changes and memorial effects as associated to ordinary derivatives.

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